The Mathematical Expertise of Mechanical Engineers – The Case of Machine Element Dimensioning

Burkhard Alpers

Department of Mechanical Engineering, Aalen University, Germany

Abstract

This contribution reports on a project that tries to capture the mathematical expertise a mechanical engineer needs in his or her daily work. We study how mechanical engineering students in their final semester work on typical tasks. The task considered in this article is concerned with machine element dimensioning in a typical gearing mechanism using an industry standard dimensioning program. One major competency we identified was "keeping track" in a large algebraic model containing a considerable number of variables. Coming to an initial design based on estimations and making variations for improvement based on mathematical and non-mathematical arguments is essential for efficient and effective work.

Introduction

The mathematical education of engineers should enable students to use mathematical concepts, models and procedures for solving daily problems in their "engineering life". For a sound educational provision we therefore need a better understanding of the role mathematics plays or might play in such daily life problems. For non-academic professions, there are several investigations of the usage of mathematics in comparison with what is taught at school (for an overview see Gainsburg (2005)). Only recently has this research been extended to engineering professions where it is much harder for a non-professional to understand the work and the role of mathematical thinking. Kent and Noss (2002) and Gainsburg (2006) investigated civil engineers and Cardella and Atman (2005, 2006) observed industrial engineering students doing their capstone projects. Using an ethnographic qualitative method of research (for engineers and students, respectively), they discovered several aspects and patterns of mathematical thinking. The work described in this contribution is concerned with the mathematical expertise of mechanical engineers. Taking an approach similar to that of Cardella and Atman, we investigate how students work on typical practical tasks. The approach and results on typical static design and mechanism design tasks can be found in Alpers (2006, 2007). Here, we report our findings when students worked on a typical machine element dimensioning task. In the next section we briefly recap the method of investigation and present the task. We then describe the principal approach of the students and discuss the findings concerning the mathematical thinking processes. Finally, we draw some conclusions for the mathematical education of mechanical engineers.

Method of Investigation

Two students in their final semester of study are given a practical task which is identified by a colleague teaching machine elements, CAD and FEM who worked in the car industry for several years. They are paid for working 100 hours on the task and they are asked to document their work process. They can seek advice from the colleague who plays the role of a group leader. The author investigates the documents of the students and conducts interviews with the students and the colleague in order to detect necessary mathematical knowledge as well as situations where a more mathematical approach might have been more efficient. The interview with the colleague also serves to check whether the work done by the students resembles the work of a junior engineer in industry and thus to recognise the limitations of the approach.

The task within this project deals with machine element dimensioning. The students are to dimension the gearing mechanism sketched in Figure 1 where an input rotational velocity of 3000 rpm (revolutions per minute) has to be transformed to approximately 1200 rpm. The input power amounts to 15 KW, the distance between the shafts is approximately 200mm and the life expectancy should exceed 10000 hrs.

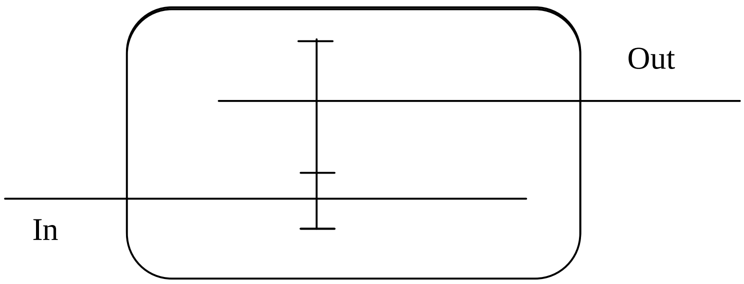

Figure 1. Sketch of Gearing Mechanism.

Approach of the Students

The essential steps for dimensioning the transformation mechanism are quite straight forward. First, the gears have to be dimensioned such that the requirements concerning transformation (3000 to 1200 rpm), distance of shafts and stiffness (see below) are met. Then, the loads for the shafts are known and the latter can be dimensioned correspondingly. Finally, the bearings for the shafts have to be sized and chosen from catalogues. Procedures and formulae for all these subtasks are written down in text books on machine elements. The students used the book by Jahnasch (2007) which is widely used at German universities and in industry as well. For performing the computations there are three modules in the machine element computation program MDesign® which is also quite widespread. The students also performed computations by hand (and pocket calculator) for the following reasons:

- for providing input data for MDesign®;
- in order to understand or repeat the computation since MDesign® just provides results;
- to check the results of MDesign® mainly for detecting errors in input values.

According to the colleague, a senior engineer would have known the computational scheme but a junior engineer would be likely to first look it up in a book. A senior engineer would perform computations by hand if the results of a program seemed to be strange.

The computational schemes can be looked up in Jahnasch (2007). In what follows we outline one major scheme that is concerned with the stiffness of the gear wheels and is typical also for other stiffness computations. There are two interesting damage situations: a tooth might break at its bottom or the tooth flank might be damaged by pressure. The computational models used for dimensioning and analysis are structurally the same. On the one hand, the allowed stress is successively computed starting with an allowed stress for the material used. Then there are five factors modelling different aspects including the geometry and the surface quality. These factors have been gained by experiments or experience and are tabulated. The product of all these quantities then is the allowed stress for the component (bottom or flank). Similarly, in order to compute the actual stress, one starts with a basic stress where the tangential force and some geometric quantities are used. Then, there are eight factors modelling influences of the geometry and of the application situation. Again, these factors have partially been gained by experiments or experience. Moreover, some of them can be computed by going into more detailed models which also contain geometric properties. The product of all these quantities is the actual stress. Finally, the quotient of both products is taken in order to compute the so called safety factor. For a non-mathematician, this is quite a complex algebraic (multiplicative) model where the quantities used are often estimated and uncertain.

Findings and Discussion

In this section we outline the essential findings concerning the use of mathematics and the necessary mathematical knowledge. We also relate the findings to the results of Gainsburg (2006) and Cardella and Atman (2005).

When working on the task and using the program the students had to have in mind the computational models for adequate dimensioning. They did not have to set up the models but the latter were given in text books and also – in a more sophisticated form – in guidelines of the German Institute for Standardisation (DIN). The models are based on many experiments and experiences and consequently the practising engineer does not know the exact reasons for all the assumptions leading to the model. Yet, he or she has to use it and work thoroughly through the sometimes lengthy computational algorithms.

The most sophisticated model outlined in the previous section is algebraic in nature. Its complexity stems mainly from the large number of variables the influence of which cannot always be seen immediately. It is essentially a multiplicative model where basic values are transformed into real values (of actual stress and bearable stress) by multiplying factors. The quotient provides the safety factor. Engineers should understand this basic algebraic structure in order to work effectively and efficiently with the model. It gives insight into the effects of potential variations. Since the model contains several different quantities representing stress, “keeping track” (Gainsburg (2006)) is a major challenge.

The values of many variables must be estimated based on the application of the mechanism which necessarily results in uncertainty. Estimation and uncertainty were identified by Cardella and Atman (2005) as well as Gainsburg (2006) as being characteristic in engineering work. Uncertainty is to be taken into account when interpreting the results of computations in the models. Safety factors must not be considered as precise quantities. When a safety factor equals 1 that does not mean that the mechanism is safe. It makes sense to compute variations for different estimations in order to check the influence on the safety factor. The requirements on safety factors cannot be justified mathematically but must be negotiated. This influence of the “social context” has already been stated by Gainsburg (2006).

When working in a model with many variables, the first problem is to get a reasonable initial design which is then transformed into an acceptable solution in several iteration cycles. Experience, rough calculations and rules of thumb play a major role in getting such an initial design. According to the colleague involved in the project there are many rough models for standard engineering tasks. When making variations of the initial design, the students often did not go back to the mathematical model but used their qualitative knowledge and the program for getting quick results. There was no optimisation in a mathematical sense but rather iterative improvement that was also detected by Gainsburg (2006) and Cardella and Atman (2005). According to the colleague, this is not always the case in real engineering life. Particularly in larger serial production, it is quite important to achieve the “final five percent” of improvement, and for this the underlying model has to be investigated thoroughly for finding additional gains. This shows that in practice the tasks and requirements differ considerably depending on the application conditions.

Beside the model for computing safety factors one also has to use a mechanical model for determining the forces in order to dimension the shafts and the bearings properly. Here, the students were insecure and made – unknowingly – several simplifications which still led to an acceptable design. Yet, in situations where the “final five percent” are important, engineers should know where they simplify and be able to change to more detailed and precise models.

The investigation of program usage showed that it is by no means possible to simply delegate the work to the machine element computation program. Often, input values have to be determined by hand in simple models. There are different program parts for the computation of gears, shafts and bearings, and to go from one part to the other requires data adaptation and computation of further data. Moreover, erroneous input is very likely to occur. Without having a good understanding of the underlying mechanical model it is hard to avoid errors.

As was emphasized by Gainsburg (2006), there were many extra-mathematical aspects that had to be taken into account. The goal of the design consists of finding a functional **and** cost-efficient solution. For the latter goal, logistic and production-related aspects play a very important role. Moreover, experience available in the company is also an essential argument for favouring certain alternatives over others. These factors can hardly be included in a mathematical optimisation model, so even if the current state of machine element computation programs will improve, one cannot expect a simple delegation of such a design task to a program.

Finally, we want to discuss the merits and limitations of our approach. The task that was suggested by the colleague seems to be quite realistic and resembles other machine element dimensioning tasks that can be seen in books on machine elements like Jahnasch (2007). By chance, the author recently supervised a diploma thesis performed by one of the students in a company producing axles and gears and similar tasks came up. Yet, there is also a limitation in that there was no "unforeseen" new application situation resulting in the necessity of model adaptation or development. The latter was recognised by Gainsburg (2006) when observing structural engineers. In our approach we cannot investigate the behaviour in such situations. There simply is no rich environment with many sources of information influencing the design process. When the students needed more information (for example, finding values for the many factors), they sometimes made (justifiable) assumptions on their own and sometimes asked the colleague acting as a mentor. Without a real application background it was also hard to decide when the work was finished, so there was no real incentive for improvement by iteration. Given the timeframe of 100 hours, the students made only very few variations. A further limitation of the method is that one can just observe behaviour similar to the one of junior engineers. We cannot capture a realistic dialogue between junior and senior engineers as was observed by Gainsburg (2006). When the students had problems we could at least ask the colleague whether the problems were specific to the students or might also be seen in real engineering life.

Beside the limitations there are also clear advantages of the method. Having the students "at hand" permits extensive interviews and gives them opportunity to show their program usage. For a mathematician or a math educator, it is not easy to understand the work of mechanical engineers. Understanding is also an iterative process. Therefore, it is very helpful when the students and the colleague are available for questions many times. This is much harder to achieve in a real company where explanations to researchers mean loss of hours billable to the client. If one does not have the opportunity to obtain a deeper understanding of the engineering task it is hardly possible to investigate whether at some stages a more mathematically oriented approach would have made the work more effective and efficient.

Conclusions for Education

One should be very careful when drawing conclusions concerning mathematical education since the observations are restricted to a few tasks. Moreover, mathematical education not only aims at providing the mathematical expertise needed in later jobs but also has to provide the mathematical concepts and procedures needed in application subjects like mechanics or control theory. Nevertheless, the results of the investigation suggest some changes or shifts in emphasis.

Modelling and working with models plays an important role for efficient work. So, setting up models and solving problems with models should be an essential part of engineering education. It is not clear, though, how the educational efforts should be distributed. Several application subjects in mechanical engineering (e.g. engineering mechanics, machine dynamics, control theory) deal mainly with developing mathematical models and solving problems. Working with such models is virtually one of the main differences between technicians/craftsmen on the one hand and engineers on the other hand. Whatever the distribution might be, it is certainly reasonable to work with models in the mathematical education and to discuss the problem of estimation and uncertainty. Methods for estimating faults resulting from insecure input data are also important. Moreover, an investigation of the influence of variables in models is important for efficient variation. Students should know the difference between qualitative improvement strategies on the one hand and mathematical optimisation models on the other hand. The merits of both approaches should be discussed in examples such that students gain a better understanding of when to use what. Mathematical application projects provide good learning opportunities in this respect. Here, estimations for application situations have to be made as is often the case in real engineering life.

The investigation has also shown the value of precise and diligent work using complex algorithms or schemes that are not fully understood. There are simply too many models and computational schemes, so for practical reasons engineers are eager to accept models set up by others particularly when they are established as guidelines or standards. So, even the often criticised traditional mathematical education solving problems using computational schemes which are only partially understood seems to have some value.

References

Alpers, B. (2006) “Mathematical Qualifications for Using a CAD program.” In S. Hibberd, L. Mustoe, eds. *Proc. IMA Conference on Mathematical Education of Engineers, Loughborough*. Engineering Council, London.

Alpers, B. (2007) “The Mathematical Expertise of Mechanical Engineers – The case of Mechanism Design.” In R. Lesh et al., eds. *Proc. ICTMA 13*. Bloomington (submitted).

Cardella, M. and Atman, C. (2005) “A qualitative study of the role of mathematics in engineering capstone projects: Initial insights.” In W. Aung et al., eds. *Innovations 2005: World innovations in engineering education and research*, Arlington (VA): International Network for Engineering Education and Research, 347-362.

Cardella, M. and Atman, C. (2005) “Engineering students’ mathematical problem solving strategies in capstone projects.” In *Proc. of the 2005 ASEE Annual Conference*.

Gainsburg, J. (2005) School mathematics in work and life: what we know and how we can learn more. *Technology in Society,* 27: 1-22.

Gainsburg, J. (2006) The mathematical modeling of structural engineers. *Mathematical Thinking and Learning,* 8: 3-36.

Kent, Ph. and Noss, R. (2002) “The Mathematical Components of Engineering Expertise: The Relationship between Doing and Understanding Mathematics.” In *Proc. of the IEE 2nd Annual Symposium on Engineering Education*, London.

Jahnasch, D. et al. (2007). *Roloff/Matek Maschinenelemente. Lehrbuch und Tabellenbuch*. 18th edition, Vieweg.