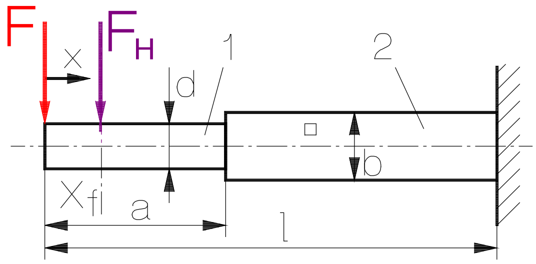

An der Stelle x_f wird die Hilfskraft F_H eingeführt.

Querkraft: $Q(x) = -F - \{x - x_f\}^0 \cdot F_H$

Biegemoment: $M_b(x) = -F \cdot x - \{x - x_f\}^1 \cdot F_H + C$

Mit der Randbedingung: $M_b(0) = C = 0$

$$\frac{\partial M_b(x)}{\partial F_H} = -\{x - x_f\}^1$$

Formänderungsarbeit infolge Biegung: $W = \frac{1}{2}\int \frac{M_b^2(x)}{EI_y(x)} dx$

$$\frac{\partial W}{\partial F_H} = \frac{1}{E}\int_0^l \frac{1}{I_y(x)} \underbrace{\left[M_b(x,F_H)\right]}_{F_H \to 0} \underbrace{\left[\frac{\partial M_b(x,F_H)}{\partial F_H}\right]}_{F_H \to 0} dx = f\,; \quad \underbrace{\left[M_b(x,F_H)\right]}_{F_H \to 0} \underbrace{\left[\frac{\partial M_b(x,F_H)}{\partial F_H}\right]}_{F_H \to 0} = +F \cdot x\{x - x_f\}^1$$

Abschnittsweise: $0 \le x \le x_f$:

$$\frac{1}{I_y(x)} \underbrace{\left[M_b(x,F_H)\right]}_{F_H \to 0} \underbrace{\left[\frac{\partial M_b(x,F_H)}{\partial F_H}\right]}_{F_H \to 0} = \frac{1}{I_{y(1)}} F \cdot x \cdot 0 = 0$$

$x_f < x \le a$:

$$\frac{1}{I_y(x)} \underbrace{\left[M_b(x,F_H)\right]}_{F_H \to 0} \underbrace{\left[\frac{\partial M_b(x,F_H)}{\partial F_H}\right]}_{F_H \to 0} = \frac{1}{I_{y(1)}} F \cdot x \cdot (x - x_f)$$

$a < x \le l$:

$$\frac{1}{I_y(x)} \underbrace{\left[M_b(x,F_H)\right]}_{F_H \to 0} \underbrace{\left[\frac{\partial M_b(x,F_H)}{\partial F_H}\right]}_{F_H \to 0} = \frac{1}{I_{y(2)}} F \cdot x \cdot (x - x_f)$$

$$f = \frac{F}{E}\left\{\frac{1}{I_{y(1)}}\left[\int_0^{x_f} 0\,dx + \int_{x_f}^{a}\left(x^2 - x_f x\right)dx\right] + \frac{1}{I_{y(2)}}\int_a^l \left(x^2 - x_f x\right)dx\right\}$$

$$f = \frac{F}{E}\left[\frac{1}{I_{y(1)}}\left(\frac{x^3}{3} - \frac{x_f x^2}{2}\right)\Bigg|_{x_f}^{a} + \frac{1}{I_{y(2)}}\left(\frac{x^3}{3} - \frac{x_f x^2}{2}\right)\Bigg|_a^l\right]$$

$$f = \frac{F}{E}\left[\frac{1}{I_{y(1)}}\left(\frac{x^3}{3} - \frac{x_f x^2}{2}\right)\Bigg|_{x_f}^{a} + \frac{1}{I_{y(2)}}\left(\frac{x^3}{3} - \frac{x_f x^2}{2}\right)\Bigg|_a^l\right]$$

$$f = \frac{F}{E}\left[\frac{1}{I_{y(1)}}\left(\frac{a^3}{3} - \frac{x_f a^2}{2} - \frac{x_f^3}{3} + \frac{x_f^3}{2}\right) + \frac{1}{I_{y(2)}}\left(\frac{l^3}{3} - \frac{x_f l^2}{2} - \frac{a^3}{3} + \frac{x_f a^2}{2}\right)\right]$$

$$f = \frac{F}{E}\left[\frac{1}{I_{y(1)}}\left(\frac{a^3}{3} - \frac{x_f a^2}{2} + \frac{x_f^3}{6}\right) + \frac{1}{I_{y(2)}}\left(\frac{l^3}{3} - \frac{x_f l^2}{2} - \frac{a^3}{3} + \frac{x_f a^2}{2}\right)\right]$$

$$I_{y(1)}=\pi\frac{d^4}{64}=12{,}56\cdot10^4\,mm^4 \qquad I_{y(2)}=\frac{b^4}{12}=52{,}1\cdot10^4\,mm^4$$

$$f=\frac{1200N}{2{,}1\cdot10^5\,N/mm^2}\left[\frac{1}{12{,}56\cdot10^4\,mm^4}\left(\frac{4^3\cdot10^6\,mm^3}{3}-\frac{4^2\cdot10^6\,mm^3}{2}+\frac{10^6\,mm^3}{6}\right)+\right.$$

$$\left.+\frac{1}{52{,}1\cdot10^4\,mm^4}\left(\frac{10^3}{3}-\frac{10^2}{2}-\frac{4^3}{3}+\frac{4^2}{2}\right)\cdot10^6\,mm^3\right]$$

$$f=\frac{1{,}2\cdot10^9\,mm}{2{,}1\cdot10^9}\left[\frac{1}{12{,}56}\left(\frac{4^3}{3}-\frac{4^2}{2}+\frac{1}{6}\right)+\frac{1}{52{,}1}\left(\frac{10^3}{3}-\frac{10^2}{2}-\frac{4^3}{3}+\frac{4^2}{2}\right)\right]$$

$$f=\frac{1{,}2mm}{2{,}1}\left(\frac{13{,}5}{12{,}56}+\frac{270}{52{,}1}\right)=3{,}6mm$$