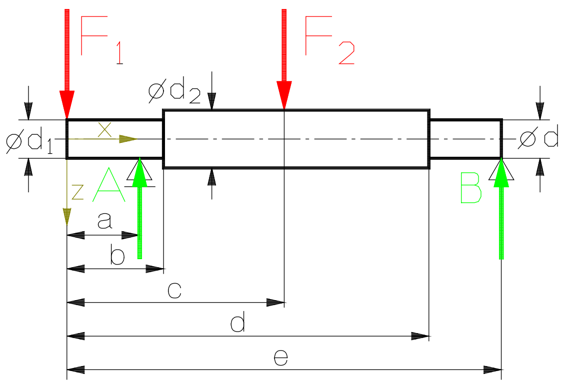

Lagerreaktionen:

$$\sum M_{i,B} = 0: \qquad A(e-a) = F_1 e + F_2 (e-c)$$

$$A = \frac{e}{e-a} F_1 + \frac{e-c}{e-a} F_2 = \frac{850}{700} F_1 + \frac{400}{700} F_2$$

$$A = \frac{17}{14} F_1 + \frac{4}{7} F_2$$

$$A = \left(\frac{17}{14} 9{,}5 + \frac{4}{7} 1{,}9\right) kN = 12{,}621 kN$$

$$\sum M_{i,A} = 0: \qquad B(e-a) = -aF_1 + (c-a)F_2$$

$$B = -\frac{a}{e-a} F_1 + \frac{c-a}{e-a} F_2 = -\frac{150}{700} F_1 + \frac{300}{700} F_2 = -\left(\frac{3}{14} F_1 - \frac{3}{7} F_2\right) = -\left(\frac{3}{14} 9{,}5 - \frac{3}{7} 1{,}9\right) kN = -1{,}22 kN$$

Satz von Castigliano:

$$f_1 = \frac{\partial W}{\partial F_1} = \int_0^e \frac{M_b(x)}{EI_y(x)} \cdot \frac{\partial M_b(x)}{\partial F_1} dx \qquad \text{mit} \qquad W_{Biegung} = \frac{1}{2} \int_0^e \frac{M_b^2(x)}{EI_y(x)} dx$$

Querkraft: $$Q(x) = -F_1 + A\{x-a\}^0 - F_2\{x-c\}^0$$

Biegemoment: $$M_b(x) = -F_1 x + A\{x-a\}^1 - F_2\{x-c\}^1 + C; M_b(0) = C = 0$$

Für die drei Belastungsabschnitte gilt:

$0 \le x \le a$: $$M_b(x, F_1) = -F_1 x \; ; \qquad \frac{\partial M_b(x, F_1)}{\partial F_1} = -x$$

$a < x \le c$: $$M_b(x, F_1) = -F_1 x + \underbrace{\left(\frac{17}{14} F_1 + \frac{4}{7} F_2\right)}_{A} x - \underbrace{\left(\frac{17}{14} F_1 + \frac{4}{7} F_2\right)}_{A} a$$

$$M_b(x, F_1) = \underbrace{\left(\frac{3}{14} F_1 + \frac{4}{7} F_2\right)}_{D} x - \underbrace{\left(\frac{17}{14} F_1 + \frac{4}{7} F_2\right)}_{A} a \; ; \qquad \frac{\partial M_b(x, F_1)}{\partial F_1} = \frac{3}{14} x - \frac{17}{14} a$$

$c < x \le e$: $$M_b(x, F_1) = -\underbrace{\left(\frac{3}{14} F_1 - \frac{3}{7} F_2\right)}_{B} (e-x); \qquad \frac{\partial M_b(x, F_1)}{\partial F_1} = -\frac{3}{14}(e-x)$$

$$f_1 = \frac{1}{EI_1}\left[\int_0^a F_1 x^2 dx + \int_a^b (Dx - Aa)\left(\frac{3}{14}x - \frac{17}{14}a\right)dx\right] + \frac{1}{EI_2}\left[\int_b^c (Dx - Aa)\left(\frac{3}{14}x - \frac{17}{14}a\right)dx - \frac{3}{14}\int_c^d B(e-x)^2 dx\right] - \frac{3}{14EI_1}\int_d^e B(e-x)^2 dx$$

$$f_1 = \frac{1}{EI_1}\left[F_1\frac{x^3}{3}\Big|_0^a + \frac{3}{14}D\frac{x^3}{3}\Big|_a^b - \frac{17}{14}Da\frac{x^2}{2}\Big|_a^b - \frac{3}{14}Aa\frac{x^2}{2}\Big|_a^b + \frac{17}{14}Aa^2x\Big|_a^b\right] + \frac{1}{EI_2}\left[\frac{3}{14}D\frac{x^3}{3}\Big|_b^c - \frac{17}{14}Da\frac{x^2}{2}\Big|_b^c - \frac{3}{14}Aa\frac{x^2}{2}\Big|_b^c + \frac{17}{14}Aa^2x\Big|_b^c + \frac{1}{14}B(e-x)^3\Big|_c^d\right] + \frac{1}{14EI_1}B(e-x)^3\Big|_d^e$$

$$f_1 = \frac{1}{EI_1}\left[\frac{F_1}{3}a^3 + \frac{D}{14}\left(b^3 - a^3\right) - \left(\frac{17}{28}Da + \frac{3}{28}Aa\right)\left(b^2 - a^2\right) + \frac{17}{14}Aa^2(b-a)\right] + \frac{1}{EI_2}\left[\frac{D}{14}\left(c^3 - b^3\right) - \left(\frac{17}{28}Da + \frac{3}{28}Aa\right)\left(c^2 - b^2\right) + \frac{17}{14}Aa^2(c-b) + \frac{B}{14}\left((e-d)^3 - (e-c)^3\right)\right] - \frac{B}{14EI_1}(e-d)^3$$

Ausrechnung:

$I = \frac{\pi}{64}d^4;$ $\quad I_1 = \frac{\pi}{64}0{,}8^4\,\mathrm{dm}^4 = 0{,}020\,\mathrm{dm}^4$; $\quad I_2 = \frac{\pi}{64}1{,}2^4\,\mathrm{dm}^4 = 0{,}102\,\mathrm{dm}^4$

$$E\,I_1 = 2\cdot 10^6\,\frac{\mathrm{kN}}{\mathrm{dm}^2}\cdot 0{,}020\,\mathrm{dm}^4 = 40212{,}4\,\mathrm{kNdm}^2$$

$$E\,I_2 = 2\cdot 10^6\,\frac{\mathrm{kN}}{\mathrm{dm}^2}\cdot 0{,}102\,\mathrm{dm}^4 = 203575{,}2\,\mathrm{kNdm}^2$$

$$D = \frac{3}{14}F_1 + \frac{4}{7}F_2 = \frac{3}{14}\cdot 9{,}5\,\mathrm{kN} + \frac{4}{7}\cdot 1{,}9\,\mathrm{kN} = 3{,}121\,\mathrm{kN}$$

$$A\,a = 12{,}612\,\mathrm{kN}\cdot 1{,}5\,\mathrm{dm} = 18{,}932\,\mathrm{kNdm}$$

$$f_1 = \frac{1mm}{402{,}12}\left[10{,}688 + (0{,}233\cdot 4{,}625) - (4{,}871\cdot 1{,}75) + (34{,}484\cdot 0{,}5)\right] + \frac{1mm}{2035{,}75}\left[(0{,}223\cdot 83{,}125) - (4{,}871\cdot 16{,}25) + (34{,}484\cdot 2{,}5) - 0{,}087\cdot(3{,}375 - 64)\right] + \frac{1mm}{402{,}12}\,\frac{1{,}223}{14}\cdot 3{,}375$$

$$f_1 = \left(\frac{20{,}436}{402{,}12} + \frac{30{,}875}{2035{,}75} + \frac{0{,}293}{402{,}12}\right)mm = (0{,}051 + 0{,}015 + 0{,}001)mm = 0{,}06672mm \approx 0{,}067mm$$