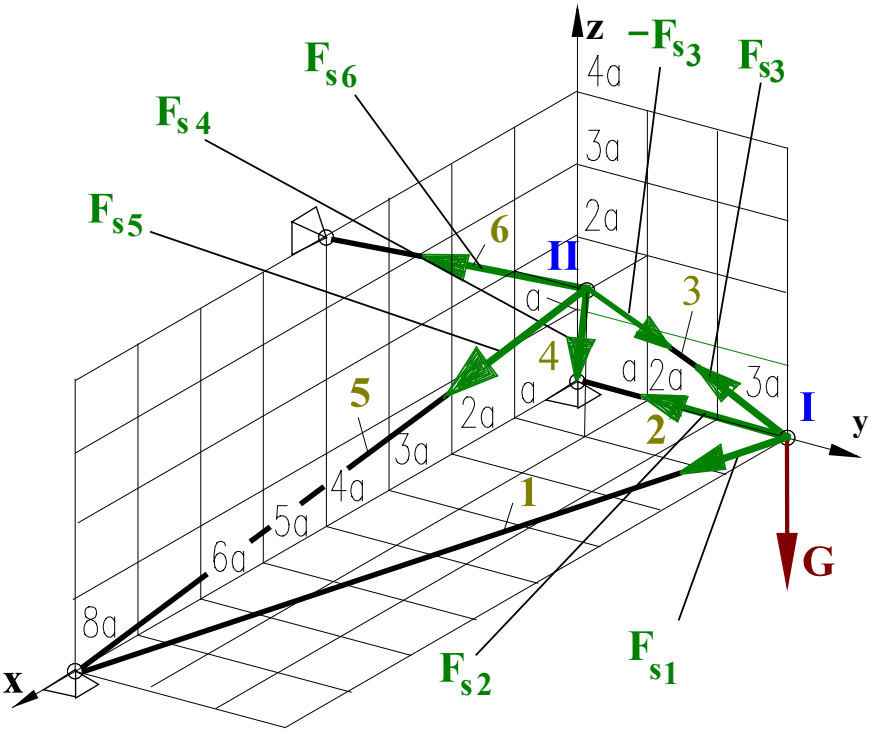

Alle Kräfte von Knotenpunkt I und II weg positiv angenommen:

$$\vec{F}_{S1} = S_1 \begin{pmatrix} 8 \\ -3 \\ 0 \end{pmatrix} \frac{1}{\sqrt{73}}$$

$$\vec{F}_{S2} = S_2 \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} \frac{1}{3}$$

$$\vec{F}_{S3} = S_3 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \frac{1}{3}$$

$$\vec{F}_{S4} = S_4 \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix} \frac{1}{\sqrt{6}}; \vec{F}_{S5} = S_5 \begin{pmatrix} 7 \\ -1 \\ -2 \end{pmatrix} \frac{1}{\sqrt{54}}; \vec{F}_{S6} = S_6 \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \frac{1}{\sqrt{14}}$$

$$\sum \vec{F}_{i,I} = 0 : S_1 \begin{pmatrix} 8 \\ -3 \\ 0 \end{pmatrix} \frac{1}{\sqrt{73}} + S_2 \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} \frac{1}{3} + S_3 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \frac{1}{3} + \begin{pmatrix} 0 \\ 0 \\ -G \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \frac{8}{\sqrt{73}} S_1 &+ 0 \cdot S_2 &+ \frac{1}{3} S_3 &= 0 \\ -\frac{3}{\sqrt{73}} S_1 &- S_2 &- \frac{2}{3} S_3 &= 0 \\ 0 \cdot S_1 &+ 0 \cdot S_2 &+ \frac{2}{3} S_3 &= G \end{aligned}$$

In Matrixschreibweise:

$$\begin{pmatrix} \frac{8}{\sqrt{73}} & 0 & \frac{1}{3} \\ -\frac{3}{\sqrt{73}} & -1 & -\frac{2}{3} \\ 0 & 0 & \frac{2}{3} \end{pmatrix} \bullet \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 48kN \end{pmatrix}$$

Koeffizientenmatrix • Lösungsvektor = rechte Seite

Lineares Gleichungssystem → Rechner

oder von Hand:

$\sum F_{iz;\ I} = 0$: $\quad S_3 \cdot 2 \cdot \frac{1}{3} - G = 0 \quad \rightarrow \quad S_3 = \frac{3}{2} G = 72\ \text{kN}$

$$\sum F_{ix,I} = 0 : S_1 \cdot 8 \cdot \frac{1}{\sqrt{73}} + \frac{1}{3} S_3 = 0 \rightarrow S_1 = -\frac{\sqrt{73}}{3 \cdot 8} S_3 = -25{,}63\,kN$$

$$\sum F_{iy,I} = 0 : -S_1 \cdot \frac{3}{\sqrt{73}} - S_2 - \frac{2}{3} S_3 = 0 \rightarrow S_2 = -39\,kN$$

$$\sum \vec{F}_{i,II} = 0 : -S_3 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \frac{1}{3} + S_4 \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix} \frac{1}{\sqrt{6}} + S_5 \begin{pmatrix} 7 \\ -1 \\ -2 \end{pmatrix} \frac{1}{\sqrt{54}} + S_6 \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \frac{1}{\sqrt{14}} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-\frac{1}{3} S_3 - \frac{1}{\sqrt{6}} S_4 + \frac{7}{\sqrt{54}} S_5 + \frac{3}{\sqrt{14}} S_6 = 0 \qquad (1)$$

$$\frac{2}{3} S_3 - \frac{1}{\sqrt{6}} S_4 - \frac{1}{\sqrt{54}} S_5 - \frac{1}{\sqrt{14}} S_6 = 0 \qquad (2)$$

$$-\frac{2}{3} S_3 - \frac{2}{\sqrt{6}} S_4 - \frac{2}{\sqrt{54}} S_5 + \frac{2}{\sqrt{14}} S_6 = 0 \qquad (3)$$

In Matrizenschreibweise:

$$\begin{pmatrix} -\frac{1}{\sqrt{6}} & \frac{7}{\sqrt{54}} & \frac{3}{\sqrt{14}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{54}} & -\frac{1}{\sqrt{14}} \\ -\frac{2}{\sqrt{6}} & -\frac{2}{\sqrt{54}} & \frac{2}{\sqrt{14}} \end{pmatrix} \cdot \begin{pmatrix} S_4 \\ S_5 \\ S_6 \end{pmatrix} = \begin{pmatrix} 24 \\ -48 \\ 48 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} S_3 \\ -\frac{2}{3} S_3 \\ \frac{2}{3} S_3 \end{pmatrix}$$

Koeffizientenmatrix • Lösungsvektor = rechte Seite

Lineares Gleichungssystem → Rechner

Oder Ausrechnung von Hand:

(3) - 2·(2): $\left(-\frac{2}{3} - \frac{4}{3}\right) S_3 + \left(\frac{2}{\sqrt{14}} + \frac{2}{\sqrt{14}}\right) S_6 = 0 \rightarrow S_6 = \frac{6}{3} S_3 \cdot \frac{\sqrt{14}}{4} = 134{,}7\,kN$

aus (1) - (2) folgt: $\left(-\frac{1}{3} - \frac{2}{3}\right) S_3 + \left(\frac{7}{\sqrt{54}} + \frac{1}{\sqrt{54}}\right) S_5 + \left(\frac{3}{\sqrt{14}} + \frac{1}{\sqrt{14}}\right) S_6 = 0$

$\rightarrow \quad S_5 = \frac{\sqrt{54}}{8} \left(S_3 - \frac{4}{\sqrt{14}} S_6 \right) = -66{,}14\,kN$

aus (1): $S_4 = \sqrt{6} \left(-\frac{1}{3} S_3 + \frac{7}{\sqrt{54}} S_5 + \frac{3}{\sqrt{14}} S_6 \right) = 51{,}44\,kN$

$F_{S1} = -25{,}63\,kN; \qquad F_{S2} = -39{,}0\,kN; \qquad F_{S3} = 72\,kN$

$F_{S4} = 51{,}44\,kN; \qquad F_{S5} = -66{,}14\,kN; \qquad F_{S6} = 134{,}7\,kN$

Negative Kräfte sind Druckkräfte, positive dagegen Zugkräfte.